



**General Certificate of Education (A-level)
June 2013**

Mathematics

MM03

(Specification 6360)

Mechanics 3

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

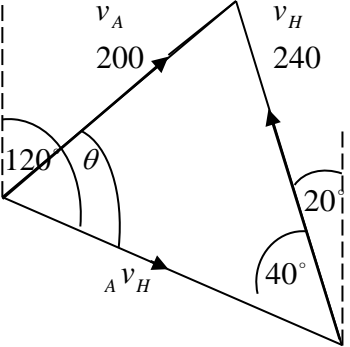
Q	Solution	Marks	Total	Comments
1	<p>Use of Impulse-momentum principle</p> $\int_{(0)}^{(T)} (3t + 1) dt = 2(5) - 2(1)$ $\left[\frac{3}{2}t^2 + t \right]_{(0)}^{(T)} = (8)$ $3T^2 + 2T - 16 = 0$ $(3T + 8)(T - 2) = 0$ <p>or $T = \frac{-2 \pm \sqrt{4 - 4(3)(-16)}}{2(3)}$</p> <p>$(T = -\frac{8}{3}$ unacceptable, not in the interval $0 \leq t \leq 3$)</p> $\underline{T = 2}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>m1</p> <p>A1</p>	6	<p>$\int_{(0)}^{(T)} (3t + 1) dt = \pm 2(5) \pm 2(1)$</p> <p>Condone sign error for M1 A1 for all correct</p> <p>Correct integration, PI by the correct quadratic</p> <p>Correct use of correct limits and rearrangement</p> <p>Solution of their quadratic, correct attempt needed</p>
	Total		6	
2	<p>$[P] = MLT^{-2} \cdot L \cdot T^{-1} = ML^2T^{-3}$</p> <p>$[mgv \sin \theta] = M \cdot LT^{-2} \cdot LT^{-1} = ML^2T^{-3}$</p> <p>$[Rv] = MLT^{-2} \cdot LT^{-1} = ML^2T^{-3}$</p> <p>$\left[\frac{1}{2}mv^3 \frac{\sin \theta}{h} \right] = M \cdot L^3T^{-3} \cdot L^{-1} = ML^2T^{-3}$</p> <p>The formula is dimensionally consistent</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>E1</p>		<p>For correct unsimplified dimensions of quantities</p> <p>All simplifications correct</p> <p>Dependent on the last B1</p>
	Total		6	

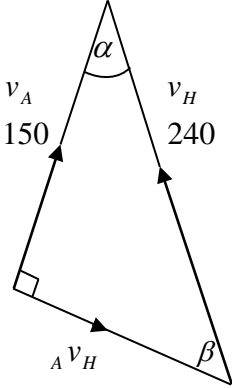
Q	Solution	Marks	Total	Comments
3(a)	$x = ut \cos \theta$	M1	6	Condone + g for M1 Elimination of t , condone + g for m1 OE in terms of $x, u, g, \tan \theta$
	$t = \frac{x}{u \cos \theta}$	A1		
	$y = -\frac{1}{2}gt^2 + ut \sin \theta$	M1		
	$y = -\frac{1}{2}gt^2 + ut \sin \theta$	A1		
	$y = -\frac{1}{2}g\left(\frac{x}{u \cos \theta}\right)^2 + u\left(\frac{x}{u \cos \theta}\right)\sin \theta$	m1		
	$y = -\frac{gx^2}{2u^2 \cos^2 \theta} + \frac{x \sin \theta}{\cos \theta}$			
	$y = -\frac{gx^2}{2u^2}(1 + \tan^2 \theta) + x \tan \theta$	A1		
(b)(i)	$0.5 = -\frac{9.8(5)^2}{2(8)^2}(1 + \tan^2 \theta) + 5 \tan \theta$	M1	5	Correctly substituting for x, y, u and g into their equation of trajectory All correct, condone decimal approximation. OE exact quadratic in $\tan \theta$ PI by the values of $\tan \theta$
	$245 \tan^2 \theta - 640 \tan \theta + 309 = 0$	A1		
	$\tan \theta = \frac{640 \pm \sqrt{(-640)^2 - 4(245)(309)}}{2(245)}$	m1		
	$\tan \theta = 1.973(004), 0.6392(41)$ $\theta = 63.12^\circ, 32.58^\circ$ $\theta = 63.1^\circ, 32.6^\circ$	A1		
(ii)	$\dot{y} = -9.8\left(\frac{5}{8 \cos 63.1^\circ}\right) + 8 \sin 63.1^\circ$ OE	M1	4	Condone +9.8 for M1. PI by correct angle in a statement Have to see “horizontal” or “vertical” or diagram
	$(\dot{y} = -6.4035)$			
	$\dot{x} = 8 \cos 63.1^\circ$ $(\dot{x} = 3.6195)$	M1		
	$\tan^{-1} \frac{6.4(035)}{3.6(195)} (= 61^\circ)$ OE	m1		
	Direction : 61° to the horizontal or 29° to the vertical	A1		

Q	Solution	Marks	Total	Comments
3(b)(ii)	<p>Alternative:</p> $\frac{dy}{dx} = -\frac{2gx}{2u^2}(1 + \tan^2 \theta) + \tan \theta$ $= -\frac{2 \times 9.8 \times 5}{2 \times 8^2}(1 + \tan^2 63.1^\circ) + \tan 63.1^\circ$ $= -1.7692$ $\tan^{-1}(-1.7692) = -60.52368^\circ$ Direction: 61° to the horizontal or 29° to the vertical	(M1) (A1) (m1) (A1)		
(c)	The ball is a particle, or No air resistance, or The ball does not spin	B1	1	
Total			16	
4(a)	$m(4u) + 3m(2u) = mv_A + 3mv_B$ $\frac{v_B - v_A}{4u - 2u} = e$ $\left(\begin{array}{l} v_A + 3v_B = 10u \\ v_B - v_A = 2ue \\ 4v_B = 2ue + 10u \end{array} \right)$ $v_B = \frac{u}{2}(e + 5)$ $\left(v_A = \frac{u}{2}(e + 5) - 2ue \right)$ $v_A = \frac{u}{2}(-3e + 5)$	M1 A1 M1 A1 A1 A1	6	M1 for four correct momentum terms with any signs. A1 for all correct M1 for correct terms for any signs, A1 for all correct. OE, simplified OE, simplified
(b)	$e \leq 1 \Rightarrow v_B \leq \frac{u}{2}(1 + 5)$ $\Rightarrow v_B \leq 3u$	M1 A1	2	Use of $e \leq 1$ (OE) needed FT their v_B
(c)	$(I =) 3m \cdot \frac{u}{2} \left(\frac{2}{3} + 5 \right) - 3m \cdot 2u$ $= \frac{5mu}{2} \text{ or } 2.5mu$	M1 A1F A1	3	M1 for a difference of two momentums FT their velocity from part (a) A1F for their 'Final B – Initial B'
Total			11	

Q	Solution	Marks	Total	Comments
5(a)	$\perp \text{ to plane } y = ut \sin \alpha - \frac{1}{2}gt^2 \cos \theta$ $y = ut \sin \alpha - \frac{1}{2}gt^2 \cos \theta$ $uT \sin \alpha - \frac{1}{2}gT^2 \cos \theta = 0$ $u = \frac{Tg \cos \theta}{2 \sin \alpha}$	M1 A1 m1 A1	4	For M1, $\sin \alpha$ and $\cos \theta$ must be in the correct terms but accept $+g$. Accept $+g$ for m1. OE
(b)	$t \text{ or } T = \frac{2u \sin \alpha}{g \cos \theta}$ $\parallel \text{ to plane } x = ut \cos \alpha + \frac{1}{2}gt^2 \sin \theta$ $x = ut \cos \alpha + \frac{1}{2}gt^2 \sin \theta$ $(\overline{OP} =) u \left(\frac{2u \sin \alpha}{g \cos \theta} \right) \cos \alpha + \frac{1}{2}g \left(\frac{2u \sin \alpha}{g \cos \theta} \right)^2 \sin \theta$ $\left(= \frac{2u^2 \sin \alpha \cos \alpha}{g \cos^2 \theta} + \frac{2u^2 \sin^2 \alpha \sin \theta}{g \cos^2 \theta} \right)$ $= \frac{2u^2 \sin \alpha (\cos \alpha \cos \theta + \sin \alpha \sin \theta)}{g \cos^2 \theta}$ $= \frac{2u^2 \sin \alpha \cos(\alpha - \theta)}{g \cos^2 \theta}$	B1 M1 A1 m1 m1 A1	6	For M1, $\cos \alpha$ and $\sin \theta$ must be in the correct terms but accept $-g$. Elimination of t substituting their expression into their equation for x . OE single correct fraction in factorised form AG Sight of the above line needed
Total			10	

Q	Solution	Marks	Total	Comments
6	(Let $v_B = a\mathbf{i} - b\mathbf{j}$)			
	$\frac{a}{b} = \frac{3}{2}$	M1		Allow sign error
	$\frac{a}{b} = \frac{3}{2}$	A1		OE
	(Squares are smooth \Rightarrow j component \Rightarrow) $b = 3$	B1		
	$a = \frac{9}{2}$	A1	4	AG
	$\left(v_B = \frac{9}{2}\mathbf{i} - 3\mathbf{j}\right)$			
	(b) (C.L.M. along the line of centres:)			
	$4(4) - 2(2) = 4(v_A) + 2\left(\frac{9}{2}\right)$	M1		OE, No sign errors
	$v_A = \frac{3}{4}$	A1		
	(Restitution along the line of centres:)			
$e = \frac{-\frac{3}{4} + \frac{9}{2}}{4 + 2}$ OE	M1 A1		M1 for correct terms, A0 for sign error	
$e = \frac{5}{8}$	A1	5		
(c) (I = Change in momentum of B along the line of centres)				
$= 2\left(\frac{9}{2}\mathbf{i}\right) - 2(-2\mathbf{i})$	M1		Allow sign error and missing \mathbf{i}	
$= 13\mathbf{i}$	A1		A0 for magnitude or $-13\mathbf{i}$	
Ns or kg m s^{-1}	B1	3		
	Total		12	

Q	Solution	Marks	Total	Comments
7(a)(i)	 <p> $\frac{\sin \theta}{240} = \frac{\sin 40}{200}$ $\theta = 50.47483^\circ$ or 50.5° Bearing of $v_A = 069.5^\circ$ </p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>5</p>	<p>Correct diagram with or without arrows.</p> <p>40° marked correctly, PI by correct method.</p> <p>Correct sine rule allowing their angle opposite 200 in their diagram.</p> <p>AWRT 50.5°, PI by correct bearing</p> <p>Allow 69.5°</p>
(a)(ii)	<p> $\frac{{}_A v_H}{\sin(180^\circ - 40^\circ - 50.5^\circ)} =$ $\frac{200}{\sin 40^\circ}$ or $\frac{240}{\sin 50.5^\circ}$ ${}_A v_H = 311.13408$ or 311 </p> <p> Time = $\frac{20}{311.13408}$ (= 0.0642809 hours) = 3.86 min </p>	<p>M1</p> <p>A1F</p> <p>M1</p> <p>A1F</p>	<p>4</p>	<p>Allow using their angle from part (a)(i).</p> <p>FT their angle from part (a)(i)</p> <p>PI by correct answer. Allow their ${}_A v_H$.</p> <p>3sf required</p>

Q	Solution	Marks	Total	Comments
7(b)	 <p> $\cos \alpha = \frac{150}{240}$ or $\sin \beta = \frac{150}{240}$ $\alpha = 51.3^\circ$ or $\beta = 38.7^\circ$ </p> <p>Bearing: 031.3°</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>5</p>	<p>Right-angled triangle with 240 and 150 marked.</p> <p>Correct orientation</p> <p>PI by correct bearing</p> <p>Allow 31.3°</p>
	Total		14	
	TOTAL		75	